Universidad
de Murcia

Departamento
Matemáticas

Pinceladas de análisis complejo y topología en análisis funcional

B. Cascales

Seminario de Análisis Funcional, Murcia, 26 de Abril de 2011

Para estar centrados

- La motivación.
- Una nueva tradición.
- Un poco sobre el GAF-UMU
- Las matemáticas de la charla.
- El desenlace.

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ANÁLISIS FUNCIONAL

Pinceladas de análisis complejo y topología en análisis funcional

RESUMEN: En esta charla se presentarán, de forma asequible para alumnos de licenciatura en matemáticas, los resultados del reciente trabajo titulado *"A Bishop-Phelps-Bollobás type theorem for uniform algebras"* (de B. Cascales, A. J. Guirao and V. Kadets). Se prestará especial atención a las ideas de análisis complejo (transformaciones conformes, teorema de Riemann, álgebra del disco) y topología (lema de Uryshon, fragmentabilidad) que se utilizan para demostrar un resultado de optimización de operadores entre espacios de Banach, a saber: *"muchos operadores T entre espacios de Banach pueden aproximarse por otros operadores S que alcanzan la norma con la propiedad adicional de que los puntos donde T casi alcanza su norma y S alcanza la suya también están próximos"*. Esta charla estará asistida por cálculos con ordenador y se dará una muestra de cómo el ordenador puede ayudar a conjeturar un resultado en análisis complejo y funcional.

La motivación. . .

. . . “cerveza” esta noche. dónde iremos?

La nueva tradición que empezamos hoy. . .

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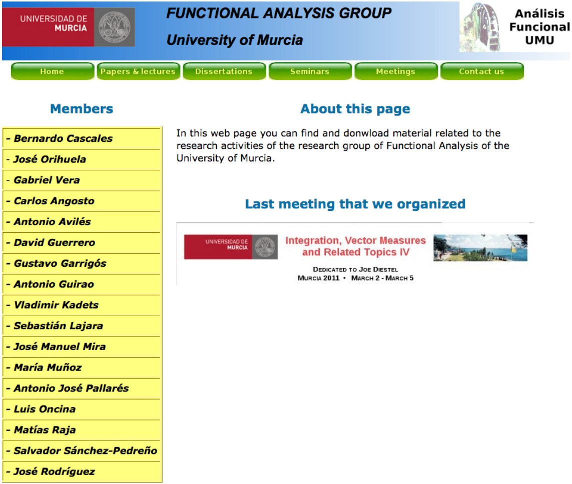
. . . el que habla se presenta.

Un poco sobre GAF-UMU. . .



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Un poco sobre GAF-UMU...

1. Crea y prepara material para las clases: Gabriel, José Manuel, Salva, etc.

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Un poco sobre GAF-UMU...

1. Crea y prepara material para las clases: Gabriel, José Manuel, Salva, etc.
2. Investiga y publica. Logra buena financiación en proyectos competitivos (prácticamente el 100 por 100 de lo que se pide)

Hay proyectos regionales y europeos dirigidos por A. Avilés, A. J. Guirao, J. Rodríguez, etc

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Un poco sobre GAF-UMU. . .

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La interacción entre teoría de la medida, topología y análisis funcional.
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DEPARTAMENTO de MATEMÁTICAS

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ANÁLISIS FUNCIONAL

Límites puntuales de funciones holomorfas y el Teorema de Runge

RESUMEN: Mientras que las funciones holomorfas suelen tener un "buen comportamiento" cuando la convergencia es uniforme o uniforme sobre compactos, surgen patologías cuando la convergencia es sólo puntual. Estudiamos dichas patologías con el Teorema de Runge y se añaden algunas condiciones para obtener una buena convergencia.

RICARDO ALCAÑIZ FRUTOS
CARLOS ALBALADEJO PADILLA

GAF-UMU y la Red de Análisis Funcional y Aplicacions

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Here you may find a list of members of the network, information regarding research groups, PhD Thesis, news and links of interest related to Functional Analysis and its Applications.

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30						

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GAF-UMU y la Escuela de la Red de Análisis Funcional y Aplicacions

Gracias a mis colegas y alumnos por el trabajo en el GAF-UMU, con mención muy especial para mi amigo Pepe Orihuela.

Metrizability of precompact subsets in (LF)-spaces

by

B.Cascales and J.Orihuela

SYNOPSIS. In this paper we prove that every precompact subset in any (LF)-space has a metrizable completion. As a consequence every (LF)-space is angelic and in this way the answer to a question posed by K.Floret ,[3], is given. Some contributions to the general problem of regularity in inductive limits posed by K.Floret,[3], are also given. Particularly, extensions of well known results of H.Neuss and M.Valdivia are provided in the general setting of (LF)-spaces. It should also be noted that our results are true for inductive limits of an increasing sequence of metrizable spaces.

1. Introduction and notations.

The vector spaces we shall use here are defined over the field $\mathbb{K}$ of real or complex numbers. The word "space" means "separated locally convex space" (briefly l.c.s.). For a space $E[\mathcal{T}]$ we denote by E' its topological dual and by $\hat{E}[\hat{\mathcal{T}}]$ its completion. If A is a bounded and absolutely convex subset in a space E, E_A is the linear hull of A endowed with the norm given by the gauge of A. A is called a Banach disc when E_A is a Banach space. A sequence (subset) is said to be Mackey-convergent (Mackey-precompact) if there is a bounded and absolutely convex subset A of E such that the sequence (subset) is contained in E_A and convergent (precompact) in this space. If A can be taken a Banach disc in the former definition the sequence (subset) is called fast convergent (fast precompact). A space E has the Mackey convergence property if every convergent sequence is Mackey-convergent and it has the strict Mackey property for precompact if, given any precompact subset B of E, there is a bounded and absolutely convex subset A of E such that B is contained in A and the topology of E_A coincides on B with the topology of E. Standard references for notations and concepts are [5] and [6].

Let E be the union of an increasing sequence $E_1 \hookrightarrow E_2 \hookrightarrow \ldots \hookrightarrow E_n \hookrightarrow \ldots$ of spaces. Let $\mathcal{T}_n$ be the topology of E_n and $E_n[\mathcal{T}_n] \hookrightarrow E_{n+1}[\mathcal{T}_{n+1}]$ continuous, n=1,2,... We denote by $E[\mathcal{T}] = \varinjlim E_n[\mathcal{T}_n]$ the inductive limit of the sequence $\{E_n[\mathcal{T}_n]: n=1,2,\ldots\}$. If every $E_n[\mathcal{T}_n]$ is metrizable we shall say that $E[\mathcal{T}]$ is a LMet-space. Let us recall that all the spaces we are dealing with are Hausdorff.

Let E be a space and $\mathcal{P}(E)$ the family of all the parts of E. E is a quasi-Suslin space, [10], if there is a mapping T from a Polish space X into $\mathcal{P}(E)$ satisfying

(a) $\bigcup\{Tx: x \in X\} = E$

El primer artículo con Pepe en la revista.

Proceedings of the Royal Society of Edinburgh, **102A**, 000–000, *1986*

Metraizability of precompact subsets in (*LF*)-spaces

B. Cascales and **J. Orihuela**
Departamento de Análisis Matemático, Facultad de Mathemáticas, Universidad de Murica, 30.001-Murcia, Spain

(MS received 3 December 1985. Revised MS received 17 March 1986)

Synopsis

In this paper we prove that every precompact subset in any (*LF*)-space has a metrizable completion. As a consequence every (*LF*)-space is angelic and in this way the answer to a question posed by K. F. Floret [3] is given. Some contributions to the general problem of regularity in inductive limits posed by K. Floret [3] are also given. Particularly, extensions of well-known results of H. Neuss and M. Valdivia are provided in the general setting of (*LF*)-spaces. It should also be noted that our results hold for inductive limits of an increasing sequence of metrizable spaces.

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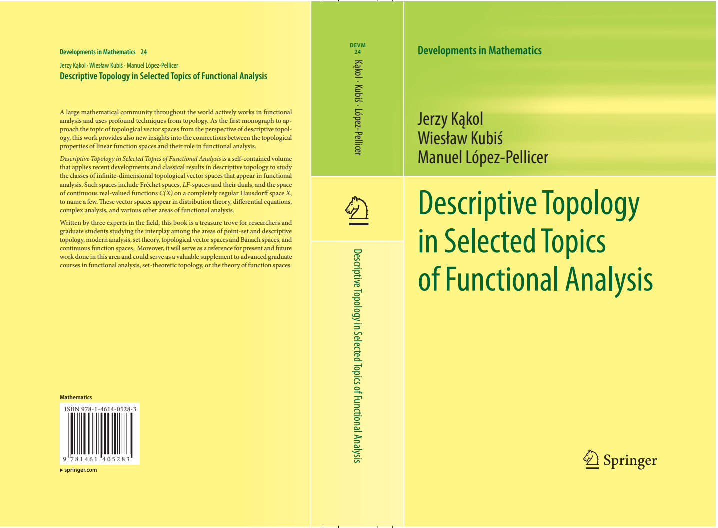
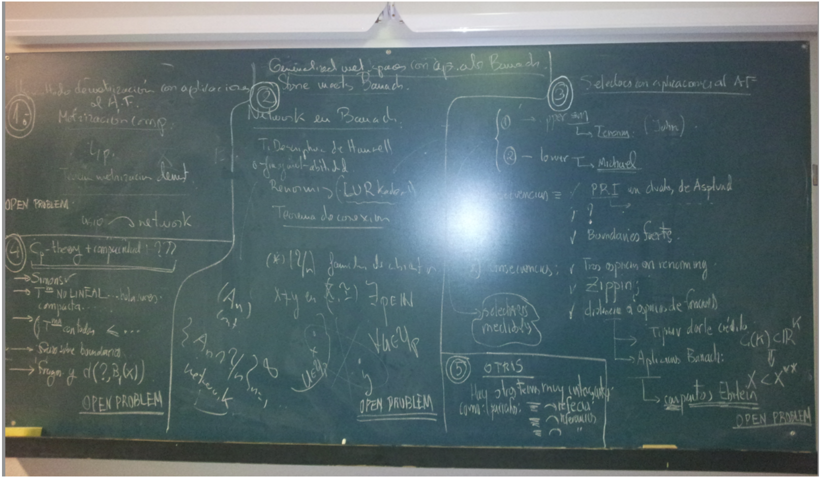
Nuestros pecados de juventud vistos por otros.



El penúltimo artículo con Pepe aún sin acabar.



PEPE HAY QUE TERMINARLO YA!.

A biased view of topology as a tool in functional analysis

B. Cascales and J. Orihuela

Abstract Each chapter should be preceded by an abstract (10–15 lines long) that summarizes the content. The abstract will appear *online* at www.SpringerLink.com and be available with unrestricted access. This allows unregistered users to read the abstract as a teaser for the complete chapter. As a general rule the abstracts will not appear in the printed version of your book unless it is the style of your particular book or that of the series to which your book belongs.

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1 Introduction

A Bishop-Phelps-Bollobás type theorem for uniform algebras

What kind of problem are we going to talk about?

A BISHOP-PHELPS-BOLLOBAS TYPE THEOREM FOR UNIFORM ALGEBRAS

B. CASCALES, A. J. GUIRAO AND V. KADETS

1. INTRODUCTION

Mathematical optimization is associated to maximizing or minimizing real functions. James's compactness theorem [17] and Bishop-Pehlps's theorem [5] are two landmark results along this line in functional analysis. The former characterizes reflexive Banach spaces X as those for which continuous linear functionals $x^* \in X^*$ attain their norm in the unit sphere S_X. The latter establishes that for any Banach space X every continuous linear functional $x^* \in X^*$ can be approximated (in norm) by linear functionals that attain the norm in S_X. This paper is concerned with the study of a strengthening of Bishop-Phelps's theorem that mixes ideas of Bollobás [6] –see Theorem 3.1 here– and Lindenstrauss [21] –who initiated the study of the Bishop-Phelps property for bounded operators between Banach spaces.

The problem for $x^*: X \to \mathbb{R}$ form and $T: X \to Y$ operator

$$\|x^*\| = \sup\{|x^*(x)| : \|x\| = 1\} \overset{\text{not always}}{=} \max\{|x^*(x)| : \|x\| = 1\}$$

$$\|T\| = \sup\{\|T(x)\| : \|x\| = 1\} \overset{\text{not always}}{=} \max\{\|T(x)\| : \|x\| = 1\}$$

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$$\|T\| = \sup\{\|T(x)\| : \|x\| = 1\} \overset{\text{not always}}{=} \max\{\|T(x)\| : \|x\| = 1\}$$

What can do it about it?

Our paper is devoted to showing that Asplund operators with range in a uniform Banach algebra have the Bishop-Phelps-Bollobás property, *i.e.*, they are approximated by norm attaining Asplund operators at the same time that a point where the approximated operator almost attains its norm is approximated by a point at which the approximating operator attains it. To prove this result we establish a Uryshon type lemma producing peak complex-valued functions in uniform algebras that are small outside a given open set and whose image is inside a symmetric rhombus with main diagonal $[0, 1]$ and small height.

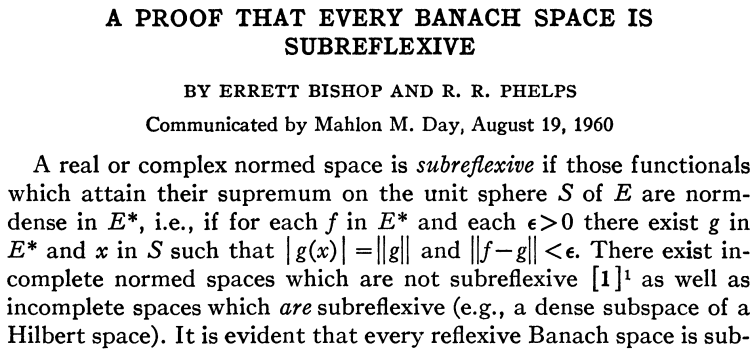
Credit to co-authors and previous work

- María D. Acosta, Richard M. Aron, Domingo García, and Manuel Maestre, *The Bishop-Phelps-Bollobás theorem for operators*, J. Funct. Anal. **254** (2008), no. 11, 2780–2799.
- R. M. Aron, B. Cascales and O. Kozhushkina, *The Bishop-Phelps-Bollobas theorem and Asplund operators*, Proc. Amer. Math. Soc. 139 (2011), no. 10, 3553–3560.
- B. Cascales, A. J. Guirao and V. Kadets, *A Bishop-Phelps-Bollobás type theorem for uniform algebras*, Enviado para publicación 19/Abril/2012
- Jerry Johnson and John Wolfe, *Norm attaining operators*, Studia Math. **65** (1979), no. 1, 7–19.
- C. Stegall, *The Radon-Nikodým property in conjugate Banach spaces. II*, Trans. Amer. Math. Soc. **264** (1981), no. 2, 507–519.

Bishop-Phelps theorem

Theorem (Bishop-Phelps, 1961)

If X is a Banach, then $\overline{\mathbf{NA}X^*} = X^*$.



A PROOF THAT EVERY BANACH SPACE IS SUBREFLEXIVE

BY ERRETT BISHOP AND R. R. PHELPS

Communicated by Mahlon M. Day, August 19, 1960

A real or complex normed space is *subreflexive* if those functionals which attain their supremum on the unit sphere S of E are norm-dense in E^*, i.e., if for each f in E^* and each $\epsilon > 0$ there exist g in E^* and x in S such that $|g(x)| = \|g\|$ and $\|f - g\| < \epsilon$. There exist incomplete normed spaces which are not subreflexive [1][1] as well as incomplete spaces which *are* subreflexive (e.g., a dense subspace of a Hilbert space). It is evident that every reflexive Banach space is sub-

The Bishop-Phelps property for operators

Definition

An operator $T : X \to Y$ is **norm attaining** if there exists $x_0 \in X$, $\|x_0\| = 1$, such that $\|T(x_0)\| = \|T\|$.

Definition (Lindenstrauss)

(X, Y) has the Bishop-Phelps Property (BPp) if every operator $T : X \to Y$ can be uniformly approximated by **norm attaining** operators.

1. $(X, \mathbb{K})$ has BPp for every X Bishop-Phelps (1961);
2. $\overline{\{T \in L(X; Y) : T^{**} \in NA(X^{**}; Y^{**})\}} = L(X; Y)$ for every pair of Banach spaces X and Y, Lindenstrauss (1963);
3. X with RNP, then (X, Y) has BPp for every Y, Bourgain (1977);
4. there are spaces X, Y and Z such that $(X, C([0,1]))$, (Y, ℓ^p) $(1 < p < \infty)$ and $(Z, L^1([0,1]))$ fail BPp, Schachermayer (1983), Gowers (1990) and Acosta (1999);
5. $(C(K), C(S))$ has BPp for all compact spaces K, S, Johnson and Wolfe, (1979).
6. $(L^1([0,1]), L^\infty([0,1]))$ has BPp, Finet-Payá (1998),

. . . Bishop-Phelps-Bollobás property

Bollobás observation

AN EXTENSION TO THE THEOREM OF BISHOP AND PHELPS

BÉLA BOLLOBÁS

Bishop and Phelps proved in [1] that every real or complex Banach space is *subreflexive*, that is the functionals (real or complex) which attain their supremum on the unit sphere of the space are dense in the dual space. We shall sharpen this result and then apply it to a problem about the numerical range of an operator.

Denote by S and S' the unit spheres in a Banach space B and its dual space B', respectively.

THEOREM 1. *Suppose* $x \in S$, $f \in S'$ *and* $|f(x)-1| \leqslant \varepsilon^2/2$ $(0 < \varepsilon < \frac{1}{2})$. *Then there exist* $y \in S$ *and* $g \in S'$ *such that* $g(y) = 1$, $\|f-g\| \leqslant \varepsilon$ *and* $\|x-y\| < \varepsilon+\varepsilon^2$.

A different way of writing BPB

THEOREM 1. *Suppose* $x \in S$, $f \in S'$ *and* $|f(x)-1| \leqslant \varepsilon^2/2$ $(0 < \varepsilon < \frac{1}{2})$. *Then there exist* $y \in S$ *and* $g \in S'$ *such that* $g(y) = 1$, $\|f-g\| \leqslant \varepsilon$ *and* $\|x-y\| < \varepsilon + \varepsilon^2$.

Corollary. . . the way is oftentimes presented

Given $\frac{1}{2} > \varepsilon > 0$, if $x_0 \in S_X$ and $x^* \in S_{X^*}$ are such that

$$|x^*(x_0)| > 1 - \frac{\varepsilon^2}{4},$$

then there are $u_0 \in S_X$ and $y^* \in S_{X^*}$ such that

$$|y^*(u_0)| = 1, \|x_0 - u_0\| < \varepsilon \text{ and } \|x^* - y^*\| < \varepsilon.$$

Bishop-Phelps-Bollobás Property for operators

Definition: Acosta, Aron, García and Maestre, 2008

(X,Y) is said to have the Bishop-Phelps-Bollobás property (BPBP) if for any $\varepsilon > 0$ there are $\eta(\varepsilon) > 0$ such that for all $T \in S_{L(X,Y)}$, if $x_0 \in S_X$ is such that

$$\|T(x_0)\| > 1 - \eta(\varepsilon),$$

then there are $u_0 \in S_X$, $S \in S_{L(X,Y)}$ with

$$\|S(u_0)\| = 1$$

and

$$\|x_0 - u_0\| < \varepsilon \text{ and } \|T - S\| < \varepsilon.$$

1. Y has *certain* almost-biorthogonal system (X,Y) has BPBp any X;
2. (ℓ^1, Y) BPBp is characterized through a condition called AHSP: it holds for Y finite dimensional, uniformly convex, $Y = L^1(\mu)$ for a σ-finite measure or $Y = C(K)$;
3. there is pair (ℓ^1, X) failing BPBp, but having BPp;
4. (ℓ^∞_n, Y) has BPBp Y uniformly convex no hope for c_0: $\eta(\varepsilon) = \eta(n,\varepsilon) \to 1$ with $n \to \infty$.

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(X,Y) is said to have the Bishop-Phelps-Bollobás property (BPBP) if for any $\varepsilon > 0$ there are $\eta(\varepsilon) > 0$ such that for all $T \in S_{L(X,Y)}$, if $x_0 \in S_X$ is such that

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4. (ℓ_n^∞, Y) has BPBp Y uniformly convex no hope for c_0: $\eta(\varepsilon) = \eta(n,\varepsilon) \to 1$ with $n \to \infty$.

PROBLEM?

No Y infinite dimensional was known s.t. (c_0, Y) has BPBP.

Our main result

Theorem (A. J. Guirao, V. Kadets and B. C. 2012)

Let $\mathfrak{A} \subset C(K)$ be a uniform algebra and $T : X \to \mathfrak{A}$ be an Asplund operator with $\|T\| = 1$. Suppose that $0 < \varepsilon < \sqrt{2}$ and $x_0 \in S_X$ are such that $\|Tx_0\| > 1 - \frac{\varepsilon^2}{2}$. Then there exist $u_0 \in S_X$ and an Asplund operator $\widetilde{T} \in S_{L(X,\mathfrak{A})}$ satisfying that

$$\|\widetilde{T}u_0\| = 1, \|x_0 - u_0\| \leq \varepsilon \quad \text{and} \quad \|T - \widetilde{T}\| < 2\varepsilon.$$

Our main result

Theorem (A. J. Guirao, V. Kadets and B. C. 2012)

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$$\|\widetilde{T} u_0\| = 1, \|x_0 - u_0\| \leq \varepsilon \quad \text{and} \quad \|T - \widetilde{T}\| < 2\varepsilon.$$

This is not what you promised: *understandable!*

Forget about $\mathfrak{A}$ and bear in mind:

- the Fourier transform $\widehat{\ }: L^1(\mathbb{R}^n) \to C_0(\mathbb{R}^n)$;
- or, $T_\varphi : A(\overline{\mathbb{D}}) \to A(\overline{\mathbb{D}})$ where $T_\varphi(f) = f \circ \varphi$ for some $\varphi \in A(\overline{\mathbb{D}})$;
- or, $T_\varphi : H^\infty(\mathbb{D}) \to H^\infty(\mathbb{D})$ where $T_\varphi(f) = f \circ \varphi$ for some $\varphi \in H^\infty(\mathbb{D})$.

The black box. . . fragmentability. . . I won't speak about it

Asplund spaces: Namioka, Phelps and Stegall

Let X be a Banach space. Then the following conditions are equivalent:

(i) X is an Asplund space, *i.e.*, whenever f is a convex continuous function defined on an open convex subset U of X, the set of all points of U where f is Fréchet differentiable is a dense G_δ-subset of U.

(ii) every w^*-compact subset of (X^*, w^*) is fragmented by the norm;

(iii) each separable subspace of X has separable dual;

(iv) X^* has the Radon-Nikodým property.

An idea of the proof for $\mathfrak{A} = C(K)$

Theorem (R. M. Aron, O. Kozhushkina and B. C. 2011)

Let $T: X \to C(K)$ be an Asplund operator with $\|T\| = 1$. Suppose that $0 < \varepsilon < \sqrt{2}$ and $x_0 \in S_X$ are such that $\|Tx_0\| > 1 - \frac{\varepsilon^2}{2}$. Then there exist $u_0 \in S_X$ and an Asplund operator $\widetilde{T} \in S_{L(X,C(K))}$ satisfying that

$$\|\widetilde{T}u_0\| = 1, \|x_0 - u_0\| \leq \varepsilon \quad \text{and} \quad \|T - \widetilde{T}\| < 2\varepsilon.$$

1. **Black box** provides a suitable open set $U \subset K$ and $y^* \in S_{X^*}$ with
$$1 = |y^*(u_0)| = \|u_0\| \text{ and } \|x_0 - u_0\| < \varepsilon.$$
2. Uryshon's lemma that applied to an arbitrary $t_0 \in U$ produces a function $f \in C(K)$ satisfying
$$f(t_0) = \|f\|_\infty = 1, f(K) \subset [0,1] \text{ and } \operatorname{supp}(f) \subset U.$$
3. $\widetilde{T}$ is explicitly defined by
$$\widetilde{T}(x)(t) = f(t) \cdot y^*(x) + (1 - f(t)) \cdot T(x)(t), x \in X, t \in K,$$
4. The suitability of U is used to prove that $\|T - \widetilde{T}\| < 2\varepsilon$.

An idea of the proof for $\mathfrak{A} = C(K)$

Theorem (R. M. Aron, O. Kozhushkina and B. C. 2011)

Let $T\colon X \to C(K)$ be an Asplund operator with $\|T\| = 1$. Suppose that $0 < \varepsilon < \sqrt{2}$ and $x_0 \in S_X$ are such that $\|Tx_0\| > 1 - \frac{\varepsilon^2}{2}$. Then there exist $u_0 \in S_X$ and an Asplund operator $\widetilde{T} \in S_{L(X,C(K))}$ satisfying that

$$\|\widetilde{T}u_0\| = 1, \|x_0 - u_0\| \leq \varepsilon \quad \text{and} \quad \|T - \widetilde{T}\| < 2\varepsilon.$$

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An idea of the proof for $\mathfrak{A} = A(\overline{\mathbb{D}})$

Theorem (A. J. Guirao, V. Kadets and B. C. 2012)

Let $T\colon X \to A(\overline{\mathbb{D}})$ be an Asplund operator with $\|T\| = 1$. Suppose that $0 < \varepsilon < \sqrt{2}$ and $x_0 \in S_X$ are such that $\|Tx_0\| > 1 - \frac{\varepsilon^2}{2}$. Then there exist $u_0 \in S_X$ and an Asplund operator $\widetilde{T} \in S_{L(X, A(\overline{\mathbb{D}}))}$ satisfying that

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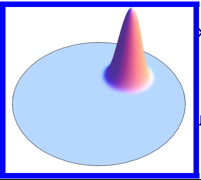

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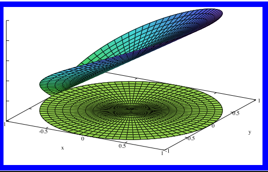

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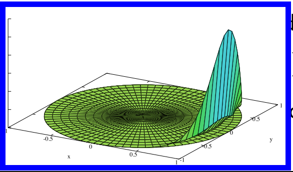

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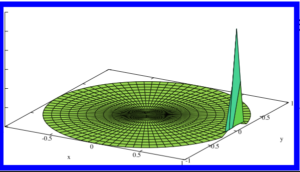

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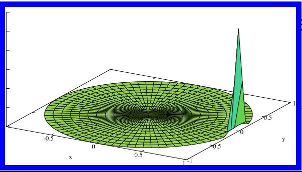

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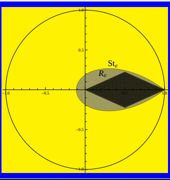

$$R_\varepsilon := \{z \in \mathbb{C} \colon |\mathrm{Re}(z) - 1/2| + (1/\sqrt{\varepsilon})|\mathrm{Im}(z)| \leq 1/2\}$$

with main diagonal $[0, 1]$, that in its turn is contained in the Stolz's domain

$$\mathrm{St}_\varepsilon = \{z \in \overline{\mathbb{D}} \colon |z| + (1 - \varepsilon)|1 - z| \leq 1\}.$$

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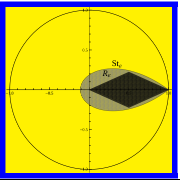

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Lemma 2.8. *Let $\mathfrak{A} \subset C(K)$ be a unital uniform algebra and Γ_0 its Choquet boundary. Then, for every open set $U \subset K$ with $U \cap \Gamma_0 \neq \emptyset$ and $0 < \varepsilon < 1$, there exist $f \in \mathfrak{A}$ and $t_0 \in U \cap \Gamma_0$ such that $f(t_0) = \|f\|_\infty = 1$, $|f(t)| < \varepsilon$ for every $t \in K \setminus U$ and $f(K) \subset R_\varepsilon$. In particular,*

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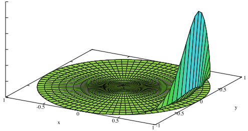

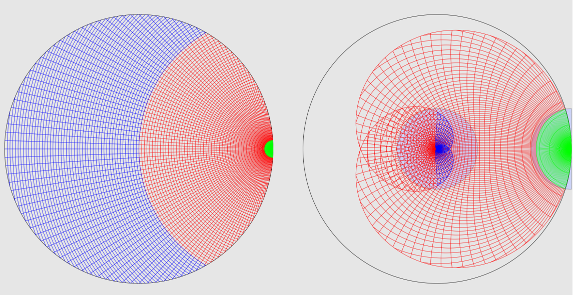

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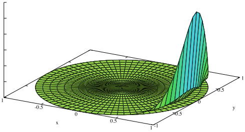

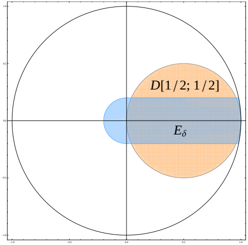

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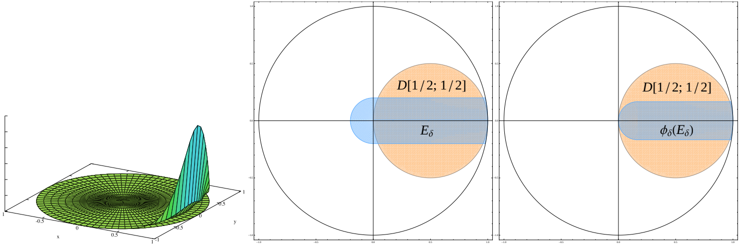

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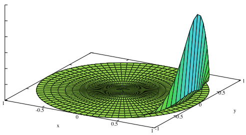

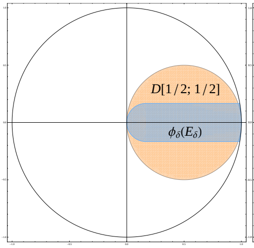

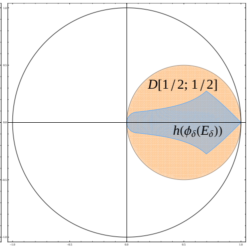

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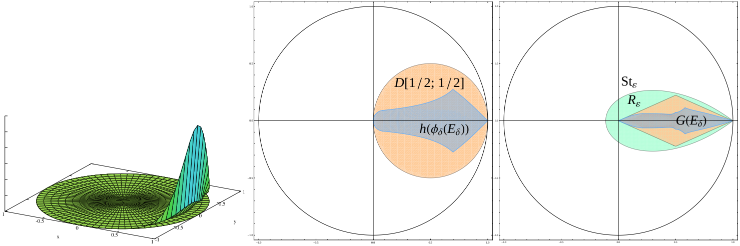

Our Uryshon type lemma is suited for calculations with a computer

Proof. Let us fix $0 < \eta < \min\{\delta/6, 1/2\}$ and $n \in \mathbb{N}$, $n > \frac{2}{\eta}$. Let us define $U_1 := U$. We shall construct inductively a collection of points $\{t_j\}_{j=1}^n$, a decreasing finite sequence $\{U_j\}_{j=1}^{n+1}$ of open subsets of U, $t_j \in U_{j+1} \cap \Gamma_0$, and functions $\{f_j\}_{j=1}^n \subset A$, satisfying for any $j \in \{1, \dots, n\}$ the following conditions:

(i) $f_j(t_j) = 1$.
(ii) $|f_j(t)| < \frac{\eta}{2}$ for $t \in K \setminus U_j$.
(iii) $|f_j(t) - 1| < \frac{\eta}{2}$ for $t \in U_{j+1}$.

Indeed, Lemma 2.1 allows us to find a norm one function $f_1 \in A$ and a $t_1 \in U_1 \cap \Gamma_0$ such that $f_1(t_1) = 1$ and $|f_1(t)| < \frac{\eta}{2}$ for $t \in K \setminus U_1$. Define

$$U_2 = \{t \in K : |f_1(t) - 1| < \eta/2\}.$$

It is clear that $t_1 \in U_2 \cap \Gamma_0$ and $U_2 \subset U_1$. Now we can apply again Lemma 2.1 to U_2 to obtain f_2, t_2 and U_3. Proceeding inductively we obtain the expected functions $f_1, \dots, f_n$, points $t_1, \dots, t_n$ and sets $U = U_1 \supset U_2 \supset \cdots \supset U_{n+1}$ with $t_j \in U_{j+1} \cap \Gamma_0$, for $j = 1, \dots, n$.

Now, we define $f := \frac{1}{n}(f_1 + \cdots + f_n)$. Given $t \in K \setminus U$, it is clear that $t \in K \setminus U_j$ for $j = 1, \dots, n$, which implies that $|f_j(t)| < \frac{\eta}{2}$ for $j = 1, \dots, n$. Therefore, we have that

$$|f(t)| < \frac{\eta}{2} \text{ for any } t \in K \setminus U. \tag{2.2}$$

Let us check that

$$\widehat{\mathrm{d}}(f(K), [0,1]) = \sup_{t \in K} \inf_{s \in [0,1]} |f(t) - s| \leq \eta. \tag{2.3}$$

...

An idea of the proof for $\mathfrak{A} = A(\overline{\mathbb{D}})$

Theorem (A. J. Guirao, V. Kadets and B. C. 2012)

Let $T: X \to A(\overline{\mathbb{D}})$ be an Asplund operator with $\|T\| = 1$. Suppose that $0 < \varepsilon < \sqrt{2}$ and $x_0 \in S_X$ are such that $\|Tx_0\| > 1 - \frac{\varepsilon^2}{2}$. Then there exist $u_0 \in S_X$ and an Asplund operator $\widetilde{T} \in S_{L(X, A(\overline{\mathbb{D}}))}$ satisfying that

$$\|\widetilde{T}u_0\| = 1, \|x_0 - u_0\| \le \varepsilon \quad \text{and} \quad \|T - \widetilde{T}\| < 2\varepsilon.$$

1. **Black box** provides a suitable open set $U \subset \overline{\mathbb{D}}$, $U \cap \mathbb{T} \neq \emptyset$ & $y^* \in S_{X^*}$ with
$$1 = |y^*(u_0)| = \|u_0\| \text{ and } \|x_0 - u_0\| < \varepsilon.$$
2. Our Uryshon type lemma applied to an arbitrary $t_0 \in U \cap \mathbb{T}$ produces a function $f \in A(\overline{\mathbb{D}})$ satisfying
$$f(t_0) = \|f\|_\infty = 1,\ f(\overline{\mathbb{D}}) \subset R_\varepsilon \text{ and } f \text{ small in } \overline{\mathbb{D}} \setminus U.$$
3. $\widetilde{T}$ is explicitly defined by
$$\widetilde{T}(x)(t) = f(t) \cdot y^*(x) + (1 - \varepsilon)(1 - f(t)) \cdot T(x)(t)$$
4. Suitability of U and the Maximum Principle in $A(\overline{\mathbb{D}})$ gives $\|T - \widetilde{T}\| < 2\varepsilon$.

Concluding remarks

- The theorem that we have proved applies in particular to the ideals of finite rank operators $\mathscr{F}$, compact operators $\mathscr{K}$, p-summing operators Π_p and of course to the weakly compact operators $\mathscr{W}$ themselves. To the best of our knowledge even in the case $\mathscr{W}(X,\mathfrak{A})$ the Bishop-Phelps property that follows is a brand new result.
- Our theorem provide unexpected examples of operators with the BBBp providing an answer to a question that appears in a paper by Acosta-Aron-García-Maestre, JFA 2008.

A lecture should finish with a problem...

A Ph. D. problem (for Olena)

What can be said for operators $T : X \to \mathfrak{A}(K, Y)$?

$$\mathfrak{A}(K, Y) := \{f : K \to Y : f \text{ continuous}, y^* \circ f \in \mathfrak{A}(K)\}.$$

A lecture should finish with a problem. . . or two

A Ph. D. problem (for Olena)

What can be said for operators $T : X \to \mathfrak{A}(K, Y)$?

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What is the true meaning of BPBp for the operators below? (Gustavo + me)

- the Fourier transform $\widehat{\ } : L^1(\mathbb{R}^n) \to C_0(\mathbb{R}^n)$;
- or, $T_\varphi : A(\overline{\mathbb{D}}) \to A(\overline{\mathbb{D}})$ where $T_\varphi(f) = f \circ \varphi$ for some $\varphi \in A(\overline{\mathbb{D}})$;
- or, $T_\varphi : H^\infty(\mathbb{D}) \to H^\infty(\mathbb{D})$ where $T_\varphi(f) = f \circ \varphi$ for some $\varphi \in H^\infty(\mathbb{D})$.

GRACIAS!

EL DESENLACE. . . la cita para

a las 9:00 p.m. *"Los Zagales"* y paga el Avilés.