



Funciones de Variable Compleja. Hoja de problemas 5.

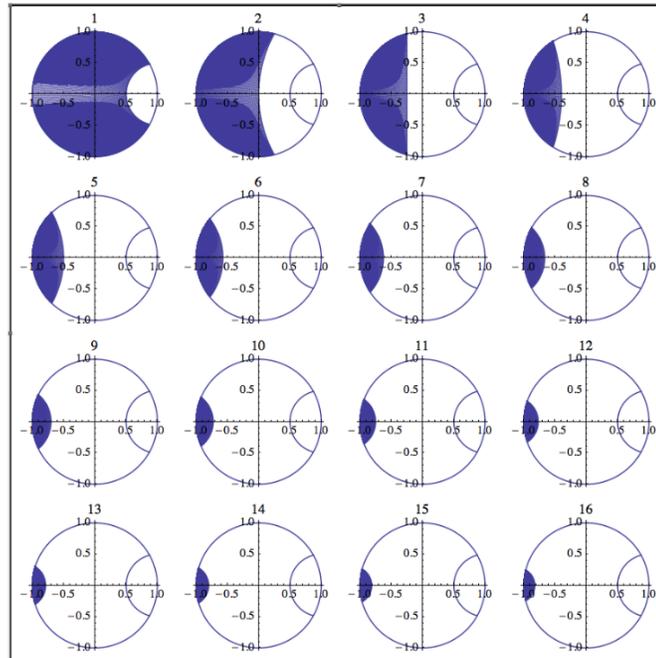
10 de Diciembre de 2012

1. Ejercicio

Take $\alpha_n := \frac{n-1}{n}$, for $n = 3, 4, \dots$ and consider the disk automorphisms given by

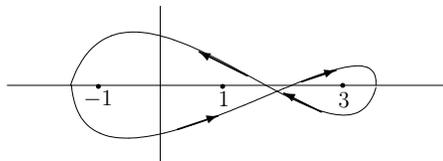
$$\varphi_n(z) := \frac{z - \alpha_n}{1 - \alpha_n z}, z \in \overline{\mathbb{D}}.$$

Note that $\varphi_n(-1) = -1$ and $\varphi_n(1) = 1$ for $n = 3, 4, \dots$. Prove, see figure below, that $\mathcal{V} := \{\varphi_n(\overline{\mathbb{D}} \setminus D(1, 1/2))\}_n$ is a basis of neighbourhoods of $-1 \in \overline{\mathbb{D}}$.



2. Ejercicio

- i) Justifique que en $\Omega = \mathbb{C} \setminus [-1, +1]$ queda determinada una raíz cúbica holomorfa f del polinomio $p(z) = (z - 1)(z + 1)^2$ que verifica $f(2) = \sqrt[3]{9}$.
- ii) Calcule las integrales $I = \int_{\gamma} f(z) dz$, $J = \int_{\Gamma} \frac{f(z)}{z - 3} dz$ donde Γ es el camino indicado en la figura



- iii) Justifique que $\lim_n r^{-n} \int_{C_\rho} z^{n-1} f(z) dz = 0$ donde $C_\rho(t) = \rho e^{it}$, $t \in [0, 2\pi]$, y $\rho > r > 1$.